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A MANUAL ON PHOTOELASTICITY

by Francis L. Ehasz

Relatively simple problems of stress distribution may be readily solved by the mathematical theory of elasticity. More complicated cases, on the other hand, must be treated experimentally, either by one of the strain-gage methods or by the photoelastic method of stress determination. The latter mode of analysis has proved very useful and is quite inexpensive once the polariscope has been acquired. Our polariscope consists essentially of a white light source, a container of water for reducing the heat, a lens for converging the beam of light on the first Nicol prism called the polarizer, an appendant quarter-plate, a lens for making the rays parallel as they strike the model, another converging lens, a second quarter-plate and Nicol prism (analyzer), and finally a system of lenses for projecting the image onto a screen or camera.

The photoelastic method is used chiefly in two-dimensional analysis, although attempts have been made to extend its applicability to three dimensions. It is based upon the temporary double-refraction induced in a beam of polarized light passing through a stressed isotropic transparent body such as glass, celluloid, bakelite, phenolite or marbellette. Celluloid is about four times as sensitive optically as glass, whereas bakelite and phenolite have, respectively, 16.5 and 21 times the optical sensitivity of glass. A newly developed photoelastic material was found in marbellette. It has a relative sensitivity of 49.3 compared to glass. The salient, beneficial properties of this material pointed out by Solakian of Columbia University, have not as yet been universally recognized. Before this can happen more must be learned regarding its annealing and preparation for photoelastic treatment. It is to be remarked that the higher the sensitivity factor, the greater the number of stress fringes for a given load. In this country bakelite has been used most extensively.

When monochromatic light passes through a doubly refracting crystal the rays emerge transformed in two respects. Each ray splits into two plane polarized rays, one of which is called the ordinary ray and the other the extraordinary ray, both vibrating in mutually perpendicular planes. It is also to be noted that these two rays travel through the crystal with different velocities. Consequently they emerge with a phase difference which depends upon the crystal length and the wave length of the light. When an element of an isotropic body is stressed, it behaves similarly as a doubly refracting crystal. All isotropic bodies subjected to two-dimensional stress within the elastic limit are optically equivalent to a large number of doubly refracting crystals whose lengths are at every point directly proportional to the difference of the principal stresses at that point and whose principal planes are parallel to the stress trajectories. It may be stated here that the two mutually perpendicular planes at a point which have only normal stresses are principal planes and the stresses acting on them are principal stresses.

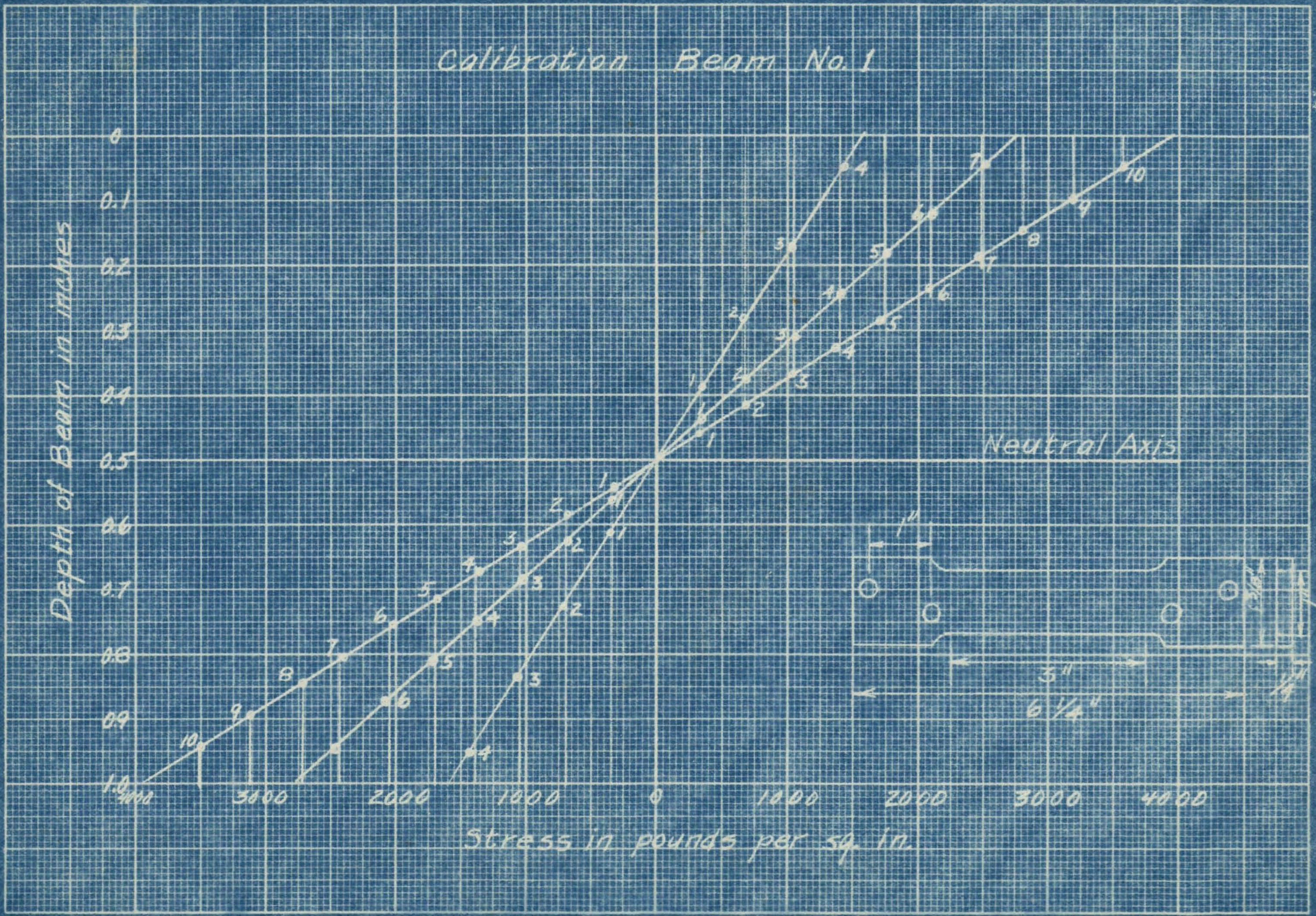
The stress-optic law is expressed as follows:

$$R = \frac{c}{a}(P-Q)t$$

where R is the retardation or phase difference between the two components of light passing through the doubly refracting crystal; a is the wave length of the light used; c is the stress retardation coefficient of the material; t represents the thickness of the model; and P and Q are the principal stresses. In words this law states that the phase difference produced between the two components of light passing through a stressed isotropic body is directly proportional to the stress difference.

Similitude of the prototype and model must be ensured in such a manner that the governing dimensions are proportionate. At the Fritz Engineering Laboratory it has been common practice to purchase No. BT-61-893 bakelite sheets, 1/4-in. thick, unpolished and suitable for photoelastic work. They may be obtained quite reasonably from the Bakelite Corporation, 247 Park Avenue, New York City. A jig-saw is instrumental in roughing the model out of the sheets; this cut should not be a finish cut, but rather one leaving about 1/16-in. on all sides. An initial annealing should precede the final filing. Annealing is best accomplished by inserting the model and its accompanying calibration beam between two glass plates which are subsequently placed in a Thelco-Electric Oven. The inner sides of the plates, as well as the model and calibration specimens, should be well oiled; any grade of machine oil will be found suitable. A temperature of eighty degrees Centigrade should be reached before the uniform cooling operation is started. An automatic clock-control arrangement enables one to lower the temperature at a uniform rate to that of the room over a period of fifteen hours. Following this heat treatment it is prudent to file and polish the specimens immediately in order to prevent the formation of internal stresses. Sandpaper and emerycloth may be used advantageously for the rough surfacing, while alumnia powder on a wet broadcloth facilitates the final polishing.

With a view of determining the fringe coefficients a calibration beam having the measurements indicated in Fig. 1 can be made from the same sheet as the model, its final thickness necessarily being the same as that of the latter. Both specimens, it is to be remarked, should be given identical treatment. A special loading device has been built for applying the constant bending moment to the beam. In Fig. 1 there are seen three straight lines, obtained for


Calibration Beam No. 1
Depth of Beam in inches
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Neutral Axis
4000
3000
2000
1000
0
1000
2000
3000
4000
Stress in pounds per sq. in.
1"
3"
6 1/4"
1/4"

different moments, indicating the stress variation with the depth. From these it is possible to arrive at the fringe coefficients.

The following determinations must be made for the complete solution of a two-dimensional problem:

1. Evaluation of (P-Q)
2. Isoclinic Lines
3. Stress Trajectories
4. Evaluation of P and Q

When the model is subjected to circular polarized light, the quarter-wave plates being in the path of the light rays, a black and white fringe photograph will result, if a filter is inserted to provide monochromatic light. It is imperative that the Nicols be crossed and have their axes at zero and ninety degrees respectively. In our case the zero position will generally be found to be about (90-16) and (360-16) or 74 and 344 degrees. The true zero values should be found, however, before each experiment, since they may vary with time and use of the apparatus. Fringe patterns, incidentally, show critical border stresses, since one of the principal stresses disappears at the boundary. It is important to determine the fringe order at each point by loading and unloading and noticing how many fringes pass the point under consideration. The black bands are arbitrarily the fringes chosen for photoelastic work. These bands show not only lines of constant

(P-Q) value but also loci of points having the same maximum shearing stress (P-Q)/2. The planes on which the latter act are at 45 degrees to the principal stress directions. Crowding of fringes show regions of stress concentration or points having external load.

Isoclinic lines are the loci of points whose principal stresses have the same inclinations. They are obtained by removing the quarter-plates, thereby effecting plane polarized light, and turning the crossed Nicols five or ten degrees at a time, depending upon the accuracy postulated. A sheet of paper having a simple diagram of the model is tacked on to a board and the latter is moved back and forth until the image of the model coincides with the sketch. Isoclinics are then readily drawn in pencil through the center of the isoclinic bands. It may be pointed out here that the quarter-plates remove the isoclinics, thus offering a means of determining whether a black spot or curve is actually an isoclinic or only a point of zero stress or a point of equal principal stresses.

Stress trajectories are obtained by drawing a series of short staccato lines having the indicated slope on each isoclinic. The trajectories should be drawn tangent to these staccato lines; Fig. 2 roughly indicates the operation. Another orthogonal set of trajectories is to be drawn perpendicular to the one shown. In addition to the isoclinics and

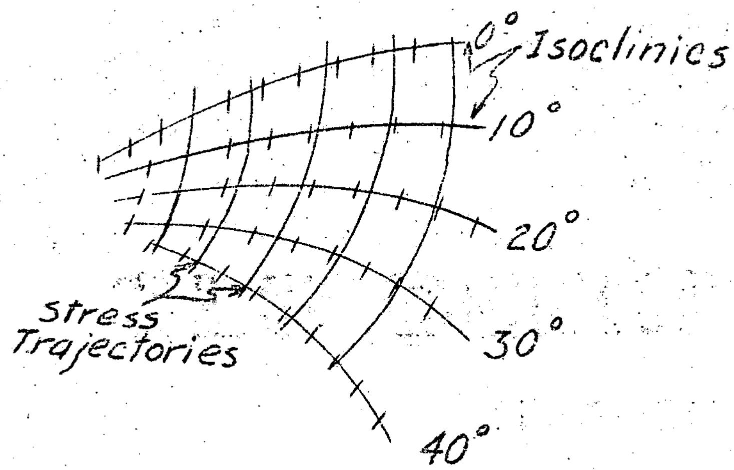


Fig. 2

stress trajectories it is necessary to sketch the fringe pattern on the same sheet before proceeding with graphical integration.

Several methods have been proposed for getting the sums of the principal stresses (P+Q) and the stresses P and Q separately; some of the more important ones are the membrane analogy, lateral extensometer method, Neuber's graphical method, Favre and Fabry's optical method, and graphical integration. The last of these has been used to a greater extent than most of the others. Papers by Filon and Weibel present the argument of this method. In brief it may be mentioned that one starts at a free boundary where one of the principal stresses is known and integrates along the stress trajectory. Considerable time and pains are involved at this stage of the analysis. Fortunately, as has been mentioned previously, the critical stresses are very often border stresses which are found directly from the fringe photograph.

With both the sum and difference of principal stresses known at each point it is possible to evaluate the principal stresses separately by simple algebra. Once these are found and the trajectories established the problem is completely solved. Normal and shearing stresses can be found from these by means of simple formulas of mechanics.

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